3rd Order Temporal Correlation Function of Pseudo-Thermal Light

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This experiment reports a nontrivial third-order temporal correlation of chaotic-thermal light in which the randomly radiated thermal light is observed to have a 6-times greater chance of being captured by three individual photodetectors simultaneously than that of being captured by three photodetectors at different times (separated by the coherent time of pseudo-thermal light), indicating a "three-photon bunching" effect. The nontrivial correlation of thermal light is the result of multi-photon interference.

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In 1956, Hanbruy-Brown and Twiss (HBT) discovered a surprising photon bunching effect of thermal light[1]. In a spatial HBT interferometer, the spatially, randomly distributed thermal light was observed to have a two-times greater chance of being captured by two individual photodetectors in a small, transverse area of correlation equal to the coherent area of the thermal radiation than that of being captured in different coherent areas. For a large angular sized thermal source the spatial correlation is effectively within a physical "point". The point-to-point near-field spatial correlation of thermal light has been utilized for reproducing nonlocal ghost images in a lensless configuration [2]. In a temporal HBT interferometer, the temporally, randomly radiated photons seem to have a two-times greater chance to be measured within a short time window which equals the coherence time of the thermal field than that of being measured in two different coherence time windows. For a natural thermal radiation source, such as the Sun, its coherence time could be as short as femtoseconds. It seems that the photons are more likely to come together and arrive at the detectors simultaneously if we try to detect them at some special space-time positions. This behavior is phenomenologically explained as "photon bunching" [3]. What is the physical cause of photon bunching? We believe this is a reasonable question to ask. It seems that there is no reason to have photons created in pairs from a thermal source. The radiation process of thermal light is stochastic. The radiated photons should be created randomly in the source, rather than bunching in pairs. In fact, this bunching effect seems even more strange for Nth-order correlation measurements of thermal light, in which more than two photo-detections are involved. Quantum theory predicts that N photons have $N!$ times greater chance of being captured within the same coherent time window of the light field than that of being captured in N different coherent time window of the light field[4].

Recently, several papers reported high order ghost imaging and ghost interference experiments by using a pseudo-thermal source[5]. In their experiments, the authors treated the electromagnetic field from the pseudo-thermal source as Gaussian random variables. Since for Gaussian random variables any moments of order higher than 2 can always be expressed in terms of the first and second order moments, they measured high order moments of pseudo-thermal light by measuring the first and second order moments with one or two CCD's, then calculated the higher order moments[6].

In this letter, we wish to report a three-photon temporal correlation measurement of pseudo-thermal light in which three photodetectors are involved in a three-fold joint measurement. The difference between our experiment and the experiments mentioned above is that in our experiment we use single photon detectors to directly measure the high order correlation function of pseudo-thermal light. We use quantum theory to describe the light field because it reveals the underlying physics and is applicable to the case when the intensity is so low that only a few photons are in the system. The experimental results show that the three individual photodetectors D_1, D_2 and D_3 have a six times greater chance of being triggered at $t_1 = t_2 = t_3$ than that of being triggered at $t_1 \neq t_2 \neq t_3$, apparently indicating that the photons are emitted in "triples" by the thermal source.

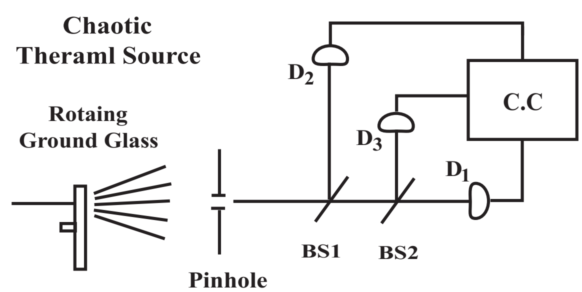

FIG. 1: Schematic setup of the experiment.

The schematic experimental setup is shown in Fig. 1. The light source is a standard pseudo-thermal source, which contains a CW laser beam, a fast rotating diffusing ground glass and a focal lens (with 25.4mm focal length). The 632.8nm laser beam is focused by a lens, onto the rotating ground glass, to a diameter of $\leq$100μm. The coherent laser beam is scattered by the fast rotating ground glass to simulate a thermal field with $\sim$0.2μs

coherence time. The coherence time of pseudo-thermal light is determined by the angular speed of the disk, the curvature of the focused laser beam and the transverse distance between the pinhole and the laser beam, the detailed discussion can be found in Ref.[8]. Effectively, the rotating ground glass produces a large number of independent point sub-sources (3 to 5 μm in diameter) with independent random phases. A pinhole with a diameter size $\sim$ 1mm is placed 800mm from the ground glass to select a small portion of the radiation within its spatial coherence area. At this distance the size of coherent area is about $\sim$ 10mm. The pseudo-thermal light passes through beam-splitters BS1 and BS2. The transmitted radiation is detected by photodetector D_1, the reflected radiations are detected by photodetectors D_2 and D_3, respectively. To simplify the discussion, we achieved equal intensities in the three paths by manipulating the transimission-reflection coefficients of the two beamsplitters and had equal distances between the light source and the three photodetectors. D_1, D_2, and D_3 are fast avalanche photodiodes working in the photon counting regime. The photo-detection response time is on the order of a few hundred picoseconds, which is much shorter than the $\sim 0.2\mu s$ coherence time of the radiation. The output pulses from D_1, D_2 and D_3 are sent to a threefold coincidence counting circuit which provides a three-photon counting histogram as a function of $t_1 - t_2$ and $t_1 - t_3$, where t_j, $j = 1, 2, 3$, is the registration time of the photo-detection event at D_1, D_2 and D_3, respectively.

The experimentally measured and simulated 3-D third-order temporal correlation functions are reported in the upper and lower parts of Fig. 2, respectively. The simulation is calculated based on Eq. (5). It is easy to see that (1) the measured correlation function is close to the simulated function, and (2) the randomly radiated “thermal photons” have much greater chance of being jointly detected in triples when $t_1 = t_2 = t_3$ than that of being detected when $t_1 \neq t_2 \neq t_3$.

From Eq. (5) we know that in order to claim the measured correlation is a third order effect the contrast between the peak and the background should be lager than 4 to 1, corresponding to a visibility larger than 60%. To compare the joint counting rate at $t_1 = t_2 = t_3$ (three-photon bunching) with the joint counting rate at $t_1 \neq t_2 \neq t_3$, a sliced “cross section” of the measured 3-D histogram is illustrated in Fig. 3. The plot is a 2-D cross section of Fig. 2(b) sliced from the top left corner to the bottom right corner. The contrast between the maximum counting rate, which occurs at $t_1 = t_2 = t_3$, and the constant background is 4.9 ± 0.25 to 1, indicating a nontrivial third-order correlation function with visibility of $\sim$ 66%, which is greater than the 2 to 1 contrast (33% visibility) of HBT. This result shows that thermal light has a much greater chance to be bunched in triples, rather than in pairs. The theoretically expected contrast is 6 to 1 (71% visibility). In addition, the single detector counting rates for D_1, D_2, and D_3 are all monitored to be constant. The reason for observing a lower visibility is due to the finite size of the detector. We expect to achieve higher visibility by simulating ideal point detectors.

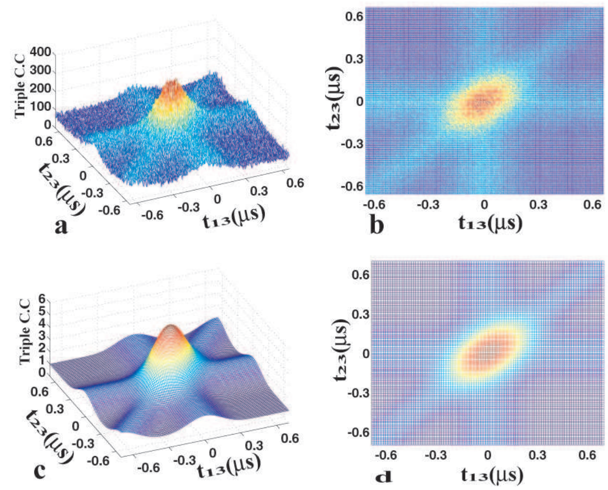

FIG. 2: (Color Online) Measured (upper, a and b) and calculated (lower, c and d) third-order temporal correlation function of thermal light. The 3-D three-photon joint detection histogram is plotted as a function of $t_{13} \equiv t_1 - t_3$ and $t_{23} \equiv t_2 - t_3$. The simulation function is calculated from Eq. (5). In addition, the single detector counting rates for D_1, D_2, and D_3 are all monitored to be constants.

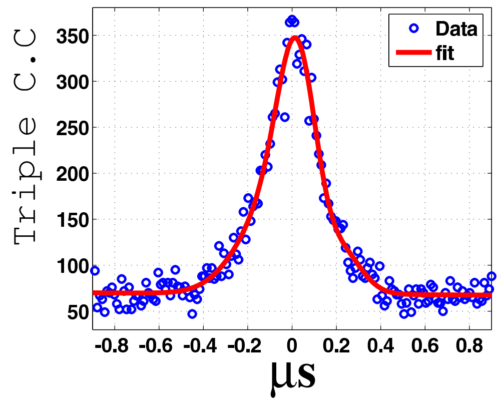

FIG. 3: (Color Online) Crosse section of the three-photon coincidence counting histogram, which is sliced from the top left corner to the bottom right corner of Fig. 2b. The contrast between the maximum counting rate and the constant background is 4.9 ± 0.25 to 1, indicating a nontrivial third-order correlation function with visibility of $\sim$ 66%.

Phenomenologically, we may name the nontrivial three-photon correlation “three-photon bunching” because in the measurement we find that the photons do have 3!-times greater chance of being jointly captured by D_1, D_2, and D_3 simultaneously than that of being jointly captured at different time(separated by coherent time of

the light field). However, does this observation really mean that three photons have 3!-times greater chance to be emitted as a bunch from the thermal source? As we have discussed earlier, we have difficulties following the photon bunching argument when N takes a large value.

In the view of quantum interference, the observed three-photon correlation of thermal radiation is the result of *three-photon interference*, which involves the nonlocal superposition of three-photon amplitudes, a nonclassical entity corresponding to different yet indistinguishable alternative ways of triggering a three-photon joint-detection event. The probability of observing a three-photon joint-detection event is calculated from the Glauber theory [9]

$$
\begin{aligned}
&G^{(3)}(t_1,t_2,t_3) \qquad (1)\\
&= \big\langle \hat{E}^{(-)}(t_1)\hat{E}^{(-)}(t_2)\hat{E}^{(-)}(t_3)\hat{E}^{(+)}(t_3)\hat{E}^{(+)}(t_2)\hat{E}^{(+)}(t_1)\big\rangle
\end{aligned}
$$

where $\langle ... \rangle$ denotes an expectation value operation based on the quantum state of the measured electromagnetic field. Thermal radiation is typically in mixed states. The generalized density operator for a chaotic-thermal field can be written as [7]

$$
\hat{\rho} = \sum_{\{n\}} p_{\{n\}}\, |\{n\}\rangle\langle\{n\}|, \qquad (2)
$$

where $p_{\{n\}}$ is the probability that the thermal field is in the state

$$
|\{n\}\rangle \equiv \prod_{\omega} |n_\omega\rangle = |n_\omega\rangle |n_{\omega'}\rangle ... |n_{\omega''...'}\rangle.
$$

Here we have simplified the problem to 1-D (temporal) with one polarization. The summation of Eq. (2) includes all possible frequency modes ω, occupation numbers n_ω for the mode ω and all possible combinations of occupation numbers for different modes in a set of $\{n\}$. The quantized field operator takes the following form

$$
\hat{E}^{(-)}_j = \int d\omega\, f(\omega)\, g(\omega, t_j, z_j)\, \hat{a}^\dagger(\omega), \qquad (3)
$$

where $f(\omega)$ is the spectral distribution function of the field, $g(\omega, t_j, z_j)$ is the Green's function which propagates each mode of the field from the source to the jth detector.

Assuming equal distances between the source and the photodetectors, i.e., $z_1 = z_2 = z_3$, the joint detection counting rate of D_1, D_2, and D_3 is calculated as [9]

$$
\begin{aligned}
&G^{(3)}(t_1,t_2,t_3)\\
&\propto \int d\omega\, d\omega'\, d\omega'' \big|f(\omega)\big|^2 \big|f(\omega')\big|^2 \big|f(\omega'')\big|^2\\
&\Big|\frac{1}{\sqrt{6}}\big[g(\omega,t_1)g(\omega',t_2)g(\omega'',t_3) + g(\omega,t_1)g(\omega'',t_2)g(\omega',t_3)\\
&+ g(\omega',t_1)g(\omega,t_2)g(\omega'',t_3) + g(\omega',t_1)g(\omega'',t_2)g(\omega,t_3)\\
&+ g(\omega'',t_1)g(\omega,t_2)g(\omega',t_3) + g(\omega'',t_1)g(\omega',t_2)g(\omega,t_3)\big]\Big|^2\\
&= \int d\omega\, d\omega'\, d\omega'' \big|f(\omega)\big|^2 \big|f(\omega')\big|^2 \big|f(\omega'')\big|^2\\
&\quad \times \Big|\frac{1}{\sqrt{3!}}\big[\sum_{\omega,\omega',\omega''} g(\omega,t_1)g(\omega',t_2)g(\omega'',t_3)\big]\Big|^2. \qquad (4)
\end{aligned}
$$

Eq. (4) is the key equation to understand the three-photon interference nature of the nontrivial third-order correlation. We notice that the probability amplitude has the similar form of the symmetrized wavefunction of three identical particles. The six terms of superposition in Eq. (4) correspond to six different yet indistinguishable alternative ways for three independent photons to trigger a three-fold joint-detection event (see Fig. 4). At $t_1 = t_2 = t_3$ (under the condition of $z_1 = z_2 = z_3$) the six amplitudes are superposed constructively, and consequently $G^{(3)}(t_1,t_2,t_3)$ achieves its maximum value when summed over these constructive interferences. On the other hand, at $t_1 \neq t_2 \neq t_3$, the six amplitudes are not superposed constructively , the cross terms equal to zero and consequently $G^{(3)}(t_1,t_2,t_3)$ achieves lower values. It is the three-photon interference that caused the three randomly distributed photons to have 6-times more chance of being captured at $t_1 = t_2 = t_3$ than that of being captured at $t_1 \neq t_2 \neq t_3$.

To simplify the calculation, taking the spectral function $f(\omega)$ a constant within the narrow bandwidth $\Delta\omega$ of the pseudo-thermal field, the normalized third-order correlation function $g^{(3)}(t_1,t_2,t_3)$ is approximately

$$
\begin{aligned}
&g^{(3)}(t_1,t_2,t_3)\\
&= 1 + \mathrm{sinc}^2[\frac{\Delta\omega(t_1-t_2)}{2}]\\
&+ \mathrm{sinc}^2[\frac{\Delta\omega(t_2-t_3)}{2}] + \mathrm{sinc}^2[\frac{\Delta\omega(t_3-t_1)}{2}] \qquad (5)\\
&+ 2\,\mathrm{sinc}[\frac{\Delta\omega(t_1-t_2)}{2}]\mathrm{sinc}[\frac{\Delta\omega(t_2-t_3)}{2}]\mathrm{sinc}[\frac{\Delta\omega(t_3-t_1)}{2}].
\end{aligned}
$$

It is easy to see that when $t_1 = t_2 = t_3$, $g^{(3)}(t_1,t_2,t_3) = 6$, the third-order correlation function achieves a maximum contrast of 6 to 1 (visibility $\sim$71%). Apparently, the randomly radiated independent thermal photons seem to have six times greater chance of being bunched in triples when we measure them simultaneously than that if we measure when $t_1 \neq t_2 \neq t_3$.

In fact, the quantum theory of light predicts that the randomly radiated independent "thermal photons" will

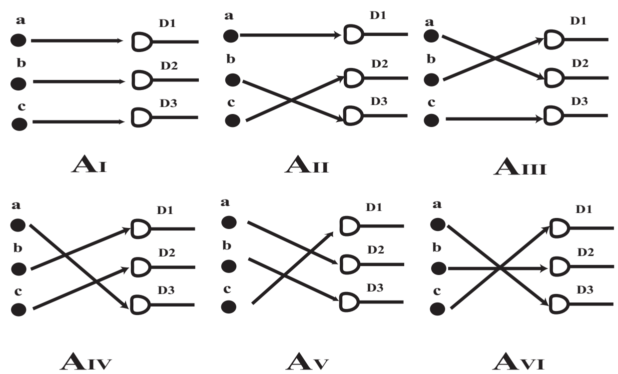

FIG. 4: Three independent photons a, b, c have six alternative ways of triggering a joint-detection event between detectors D_1, D_2, and D_3. At equal distances from the source, the probability of observing a three-photon joint-detection event at (t_1, t_2, t_3) is determined by the superposition of the six three-photon amplitudes. At $t_1 = t_2 = t_3$ six amplitudes superpose constructively. $G^{(3)}(t_1, t_2, t_3)$ achieves its maximum value by summing over these constructive interferences.

have $N!$ times greater chance of achieving "N-photon bunching" in an N-fold joint-detection of N individual photodetectors. For large N, the contrast of the Nth-order correlation may achieve $\sim$100%. The calculation is similar to that of the three-photon [4]

$$
\begin{aligned}
& G^{(N)}(t_1, ..., t_N) \\
\propto & \int d\omega \; ... \int d\omega''^{...\prime} \big|f(\omega)\big|^2 \; ... \; \big|f(\omega''^{...\prime})\big|^2 \\
& \times \Big| \frac{1}{\sqrt{N!}} \big[\sum_{\omega ... \omega''^{...\prime}} g(\omega, t_1) ... g(\omega''''^{...\prime}, t_N) \big] \Big|^2 . \qquad (6)
\end{aligned}
$$

Eq. (6) indicates that the Nth-order correlation of thermal radiation is the result of N-photon interference. It is easy to see that if all of the N-photon amplitudes are superposed constructively at a certain experimental condition, the thermal radiation will have $N!$ times greater chance to be measured under that experimental condition.

Although we have observed the nontrivial three-photon correlation with a contrast much higher than 2 to 1, and expect to have N-photon correlation with a contrast of $N!$ to 1, the quantum theory does not prevent having a constant counting rate for each of the photodetectors D_1, D_2, and D_3, respectively. In fact, the counting rate of D_1, D_2, and D_3 was monitored during the measurement and found to be constant, indicating a temporal randomly emitted and distributed stochastic emission process in the thermal source. According to the Glauber photodetection theory, the counting rate of a photodetector is proportional to the first-order self-correlation function $G^{(1)}(\mathbf{r}, t)$, which measures the probability of observing a photo-detection event at a space-time coordinate $(\mathbf{r}, t)$. For the experimental setup of Fig. 1,

$$
G^{(1)}(\mathbf{r}, t) = \langle \, E^{(-)}(\mathbf{r}, t) E^{(+)}(\mathbf{r}, t) \, \rangle = \text{constant}.
$$

In conclusion, we have observed the nontrivial 3-photon correlation of pseudo-thermal light. The photons are always randomly radiated from the thermal source. This experiment shows a nontrivial third-order temporal correlation of chaotic-thermal light in which the randomly radiated thermal light is observed to have a 6-times greater chance of being captured by three individual photodetectors simultaneously than that of being captured by the same detectors at different times (separated by the coherence time of light field).The nontrivial Nth-order correlation of thermal light can be interpreted as the result of multi-photon interference, involving the superposition of multi-photon amplitudes, a nonclassical entity corresponding to different yet indistinguishable alternative ways of triggering a joint-detection event between multiple detectors.

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